

CLAIMS

What is Claimed is:

1. In a Montgomery multiplication algorithm, a method of computing an output $xyR^{-1} \bmod m$ from input integers $m = (m_{n-1} \dots m_1 m_0)_b$, $x = (x_{n-1} \dots x_1 x_0)_b$, $y = (y_{n-1} \dots y_1 y_0)_b$, with $0 \leq x, y < m$, $R = b^n$ with $\gcd(m, b) = 1$, and $m' = -m^{-1} \bmod b$ comprising the steps of:

representing a value A in the Montgomery multiplication algorithm with a redundant notation; and

performing carry-save addition within an iterative process of the Montgomery multiplication algorithm to compute a new value of A .

2. The method of Claim 1, further comprising executing the algorithm:

a. $S \leftarrow 0$ and $T \leftarrow 0$ (where $S = (s_n s_{n-1} \dots s_1 s_0)_b$ and $T = (t_n t_{n-1} \dots t_1 t_0)_b$.)

b. For i from 0 to $(n-1)$, do the following:

b1. $u_i \leftarrow (s_0 + t_0 + x_i y_0) m' \bmod b$

b2. $(S, T) \leftarrow (S + T + x_i y + u_i m) / b$

c. $A_1 \leftarrow S + T$ and $A_2 \leftarrow S + T - m$

d. If $A_2 \geq 0$ then $A \leftarrow A_2$ else $A \leftarrow A_1$

e. Return A

3. The method of Claim 2, wherein the value of b is two.

4. The method of Claim 2, wherein step b2 of the algorithm further comprising selecting one of $y+m$, y , m , or 0 to be added to S and T .

5. The method of Claim 1, wherein said representing step further comprises replacing said value of A with two other values, wherein a sum of the other two values equals said value of A .

6. The method of Claim 1, wherein said performing step further comprises performing said carry-save addition in parallel.

7. The method of Claim 1, wherein said performing step further comprises executing said iterative process a plurality of times equal to a number of bits
5 representing said value of x .

8. A method of performing a Montgomery exponentiation of $x^e \bmod m$ wherein the notation $\text{Mont}(x, y, m)$ denotes the Montgomery multiplication algorithm $yxR^{-1} \bmod m$ from input integers $m = (m_{n-1} \dots m_1 m_0)_b$, $x = (x_{n-1} \dots x_1 x_0)_b$, $y = (y_{n-1} \dots y_1 y_0)_b$, with $0 \leq x, y < m$, $R = b^n$ with $\gcd(m, b) = 1$, and $m' = -m^{-1} \bmod b$ comprising the steps
10 of:

representing a value A in the Montgomery multiplication algorithm with a redundant notation; and

performing carry-save addition within an iterative process of the Montgomery multiplication algorithm to compute a new value of A .

9. The method of Claim 8, wherein m has s bits, e has $t+1$ bits, and $0 < x < m$, and the Montgomery exponentiation is performed according to the algorithm:

- a. Compute $R = b^s$
- b. Compute $R^*R \bmod m$
- c. Compute $z = \text{Mont}(x, R^*R \bmod m, m)$
- d. Compute $B = R \bmod m$
- e. For i from t to 0 do the following:
 - e.1 $B = \text{Mont}(B, B, m)$
 - e.2 If $e_i = 1$ then $B = \text{Mont}(B, z, m)$
- f. $B = \text{Mont}(B, 1, m)$
- g. Return B .

10. The method of Claim 8, further comprising executing the algorithm for Mont (x, y, m):

a. $S \leftarrow 0$ and $T \leftarrow 0$ (where $S = (s_n s_{n-1} \dots s_1 s_0)_b$ and $T = (t_n t_{n-1} \dots t_1 t_0)_b$.)

5 b. For i from 0 to $(n-1)$, do the following:

b1. $u_i \leftarrow (s_0 + t_0 + x_i y_0) m' \bmod b$

b2. $(S, T) \leftarrow (S + T + x_i y + u_i m) / b$

c. $A_1 \leftarrow S + T$ and $A_2 \leftarrow S + T - m$

d. If $A_2 \geq 0$ then $A \leftarrow A_2$ else $A \leftarrow A_1$

10 e. Return A

11. The method of Claim 10, wherein the value of b is two.

12. The method of Claim 10, wherein step b2 of the algorithm further comprising selecting one of $y+m$, y , m , or 0 to be added to S and T .

15 13. The method of Claim 8, wherein said representing step further comprises replacing said value of A with two other values, wherein a sum of the other two values equals said value of A .

14. The method of Claim 8, wherein said performing step further comprises performing said carry-save addition in parallel.

20 15. The method of Claim 8, wherein said performing step further comprises executing said iterative process a plurality of times equal to a number of bits representing said value of x .

16. The method of Claim 8, further comprising solving a plurality of Montgomery multiplication problems simultaneously.

17. The method of Claim 16, and further comprising controlling selection of values to be added to S and T corresponding to each said of said plurality of Montgomery multiplication problems.

5 18. An apparatus for performing a Montgomery multiplication for an output of $xyR^{-1} \bmod m$ and an input of $m = (m_{n-1} \dots m_1 m_0)_b$, $x = (x_{n-1} \dots x_1 x_0)_b$, $y = (y_{n-1} \dots y_1 y_0)_b$, with $0 \leq x, y < m$, $R = b^n$ with $\gcd(m, b) = 1$, and $m' = -m^{-1} \bmod b$, comprising:

10 a plurality of storage units, with each one of the plurality of storage units corresponding to a value of x , m , y , S and T , where $A = S + T$, wherein the corresponding storage unit for x can shift its value, and the corresponding storage units for S and T can be set to zero;

an adder configured to add the k terms from x_i times y , the k terms from u_i times m , and S and T ; and

a carry look-ahead adder configured to add any one set of values from a group including S and T , and S shifted and T shifted.

15 19. The apparatus of Claim 18, wherein for b greater than or equal to two, the adder further comprises a carry-save adder within an iterative process of the Montgomery multiplication algorithm to compute a new value of A .

20. The apparatus of Claim 19, wherein said carry-save adder further comprises a Wallace tree.

20 21. The apparatus of Claim 18, wherein b is equal to two.

22. The apparatus of Claim 21, wherein the adder further comprises a full adder configured to add input values of S , T , and any one value from a group consisting of m , y , $m+y$, zero, and $\sim m$ to produce a sum value and a carry value.

23. The apparatus of Claim 18, wherein said plurality of storage units and said added are operatively coupled to execute the algorithm:

a. $S \leftarrow 0$ and $T \leftarrow 0$ (where $S = (s_n s_{n-1} \dots s_1 s_0)_b$ and $T = (t_n t_{n-1} \dots t_1 t_0)_b$.)

5 b. For i from 0 to $(n-1)$, do the following:

b1. $u_i \leftarrow (s_0 + t_0 + x_i y_0) m' \bmod b$

b2. $(S, T) \leftarrow (S + T + x_i y + u_i m) / b$

24. The apparatus of Claim 23, wherein said carry look-ahead adder is adapted to execute the algorithm:

10 c. $A_1 \leftarrow S + T$ and $A_2 \leftarrow S + T - m$

d. If $A_2 \geq 0$ then $A \leftarrow A_2$ else $A \leftarrow A_1$

25. The apparatus of Claim 23, wherein the storage units corresponding to said value of x , y , m , $y+m$, S , and T are further adapted to hold a plurality of Montgomery multiplication problems that are solved simultaneously, and further comprising means for controlling the selection of values to be added to S and T corresponding to each said of said plurality of Montgomery multiplication problems.

26. The apparatus of Claim 25, wherein for each of said plurality of Montgomery multiply problems, the carry look-ahead adder is adapted to execute the algorithm:

20 c. $A_1 \leftarrow S + T$ and $A_2 \leftarrow S + T - m$

d. If $A_2 \geq 0$, then $A \leftarrow A_2$, else $A \leftarrow A_1$.